Orbit and escape

Worksheet 1.

You are members of a scientific team and your mission is to explore a planet, with your space ship. Once you reach it, you will orbit and take measurements about its physical quantities and chemical composition. After that you will return. The members of your team have to become experts on the universal gravitation theory.

You are assigned to learn everything about the velocity of circular trajectory as close as possible to the planet and the minimum initial velocity required to escape from the gravitational field of this planet. You can study these two magnitudes, of strategic importance for your travel, using the interactive application: "O_&_E", which is based on the so called "Newton's cannonball", a logical experiment that Newton formulated, to describe the several kinds of trajectories that an launched object can follow, caused by both the initial velocity and the gravitational force. In short, it could be summarized in the following sentences:

- *Go on top of the highest mountain of Earth, launch horizontally a projectile from that point, taking into account that the atmosphere of the Earth does not exist. If you give an orbital velocity, it will orbit the Earth endlessly. If the velocity is bigger enough, then the projectile will escape and if it is smaller it will fall on the ground.*

In the Newton's time it was just a hypothetical scenario with scientific interest but today we can send objects over the Earth's atmosphere and make them artificial satellites or interplanetary bodies, providing the appropriate velocity.

1st activity:

Set values for the mass M of the planet, its radius r and the distance of the body from the planet's surface H and define the velocity of circular orbit ($V_{circular}$). Afterwards change r and H keeping the sum R=r+H as well as the mass of the planet unchanged and find again the circular orbit velocity. Fill in the table below. You can express the mass of the planet in M_{Earth} (masses of Earth), as it is given by the application or in kilograms, multiplying by $6 \cdot 10^{24}$ kg, which is approximately the mass of Earth, expressed in kg. This is the mass of Earth we will use here, wherever required.

M=.				
	r (km)	H (km)	r+H (km)	$V_{circular}$ (m/s)
1				
2				
3				
4				

Discuss with your classmates and your teacher and formulate a sentence that covers your ascertainment. You discovered a property that derives from the Universal Gravitation Theory and characterizes the gravitational attitude of huge spherical bodies, theoretically and in a usually adequate approach as far as the real bodies are concerned.

2nd activity:

Define the velocity of circular orbit, close to the surface of the Earth. This specific velocity is named "1st cosmic velocity".

Way of work:

It is proposed to use a technique used many times by the researchers.

Let us suppose that a physical magnitude depends on N other magnitudes and we try to define this dependence. While keeping stable the values of the N-1 magnitudes we change the value of the last one. The dependence can be found, if you apply some mathematical aspects regarding the proportional and the inversely proportional quantities.

Continue in the same way to find the connection of the magnitude with another of the N magnitudes.

When you finish, you will have made a big step towards your aim, having defined the formula in the form of proportionality. In a higher level of approach, we will change the proportionality to equality.

Of course many kinds of dependences exist. However the formulas we use are covered many times by this table. Let us try to find the formula, hoping that we are "lucky"...

3rd activity:

Use the sliders to give values for M, r and H of a planet. Keep the mass stable and change r and H. A convenient way to reach a safe ascertainment is to multiply the sum r+H by 2, 3, 4 etc. Find the velocity of the circular orbit in each case and fill in the table below.

M=.				
	r (km)	H (km)	r+H (km)	$V_{circular}$ (m/s)
1				
2				
3				
4				
5				
6				
7				
8				

Proceed with the data you have recorded, and taking into consideration the mathematical assistance of the previous page and define the dependence of $V_{circular}$ on r+H. No need to fill in the whole table, you can stop as soon as you are sure you found the formula. Write it in form of proportionality and describe the way you thought.

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4th activity:
Now choose a sum r+H, keep it stable and change the mass of the planet. Fill in the table below and define the dependence of Vcircular on r+H. Again, it is not needed to fill in the whole table, you can stop as soon as you are sure you found the formula. Write it in the form of proportionality and describe the way you thought.

R = r + H =.		
	M (M_E)	$V_{circular}$ (m/s)
1		
2		
3		
4		
5		
6		
7		
8		

Now you are ready to combine the two formulas you found, to formulate a sentence that describes the way that the $V_{circular}$ is connected to the mass of the planet and the distance of its centre and to write the respective mathematical formula.

5th activity:
Find the escape velocity from the surface of the Earth (2nd cosmic velocity).

6th activity:
We try to determine the formula of the initial velocity of a body, enough to escape the

gravitational field of the planet. You can realize while running the applet, that there are a lot of such velocities, however all of them are bigger than a minimum value, which is called "escape velocity" (Vescape) and we will try to determine, as a function of M and r+H. After the previous activities you are wise enough to run the application several times, fill in the tables below and write the proportionality that connects the escape velocity to the mass of the planet and the distance of the point of launch from its centre. In order to work quickly, it is not necessary to use small values of precision.

M=.				
	r (km)	H (km)	r+H (km)	V_{escape} (m/s)
1				
2				
3				
4				
5				
6				
7				
8				

r + H =. . . . . (km)		
	M ($M_Γ$)	V_{escape}(m/s)
1		
2		
3		
4		
5		
6		
7		
8		

Evaluation

1. Write briefly what you defined and describe the method you followed.

2. Suppose you know the 1st cosmic velocity V1. What is the velocity of the circular orbit, close to the surface of a planet with mass and radian double the mass and the radian of the Earth?

3. Explain the way you will work with the applet O_&_E to find the period of a satellite that travels in circular orbit at a distance of 1600 km from the surface of the Earth and find this period.

4. Working with the application O_&_E, show that the circular orbit velocity at a specific height from the surface of a planet, is less than the escape velocity of the same height by a factor 1,4, approximately.

5. The mass of a planet is 40 times the mass of the Earth and the escape velocity from the surface of this planet is 10 times the respective velocity from the surface of the Earth. You are requested to find the radius of the planet.