Orbit and escape
Worksheet 2.

You have already found that $V_{circular} \sim [M/R]^{1/2}$ and $V_{escape} \sim [M/R]^{1/2}$.
R=r+H, the distance between the point of launch and the centre of the planet.
Let's suppose that there is a missing coefficient, which transforms each one proportionality to equality (not necessarily the same in both formulas). If it is so, then the graphs $V^2_{circular}$ vs [M/R] and V^2_{escape} vs [M/R] should have the form of a 1st degree graph and the slope of the graph will give us the coefficient we are searching for.

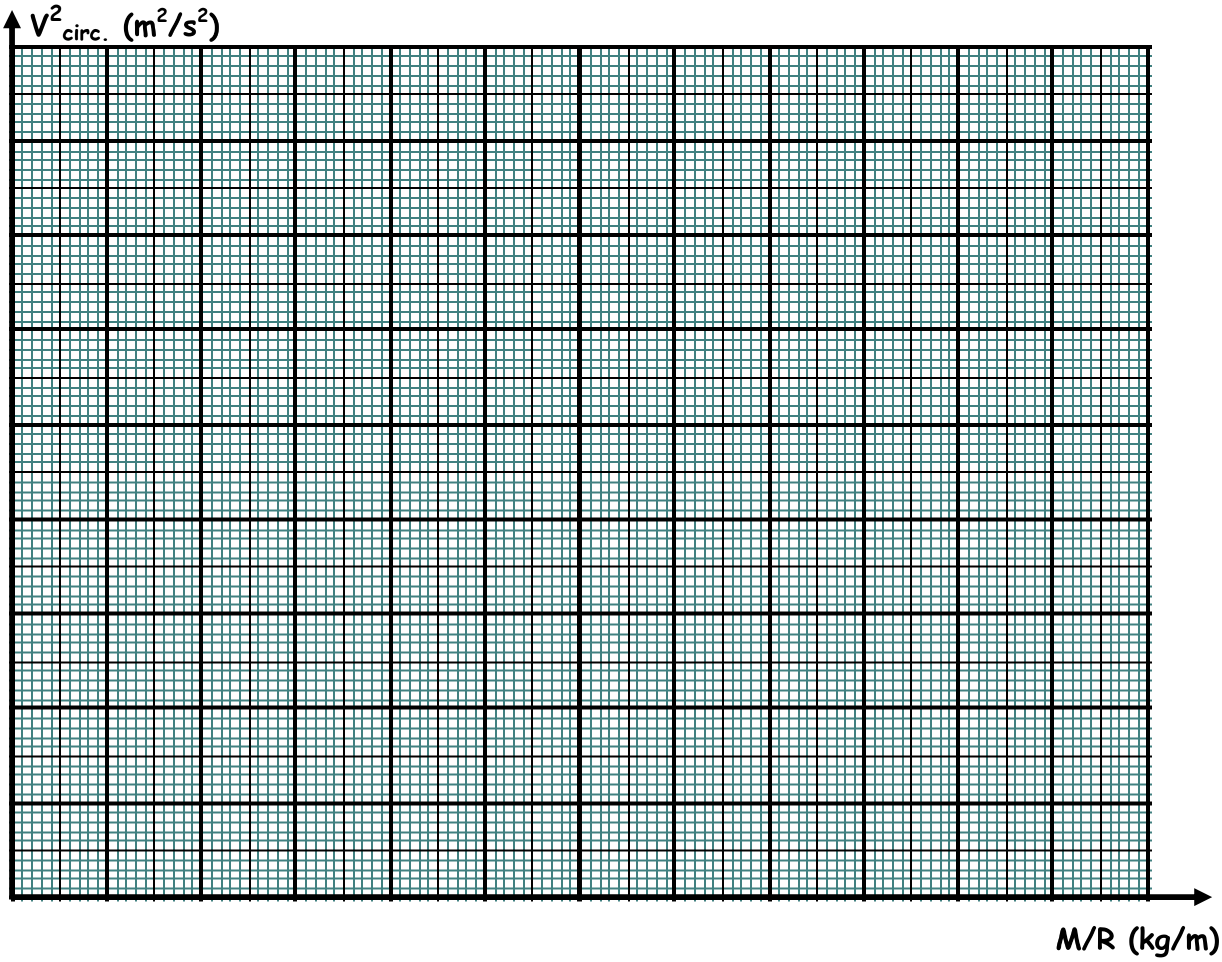
1st activity:

After all above, suppose that the complete formula has the form: $V^2_{circular} = C_1[M/R]$. C_1 is the under research factor. Compare the units of V^2 and M/(R) and define the units of the C_1. Find a table with universal constants and look for a constant with these units. Such a constant, if existing, would be a good candidate for our equations.

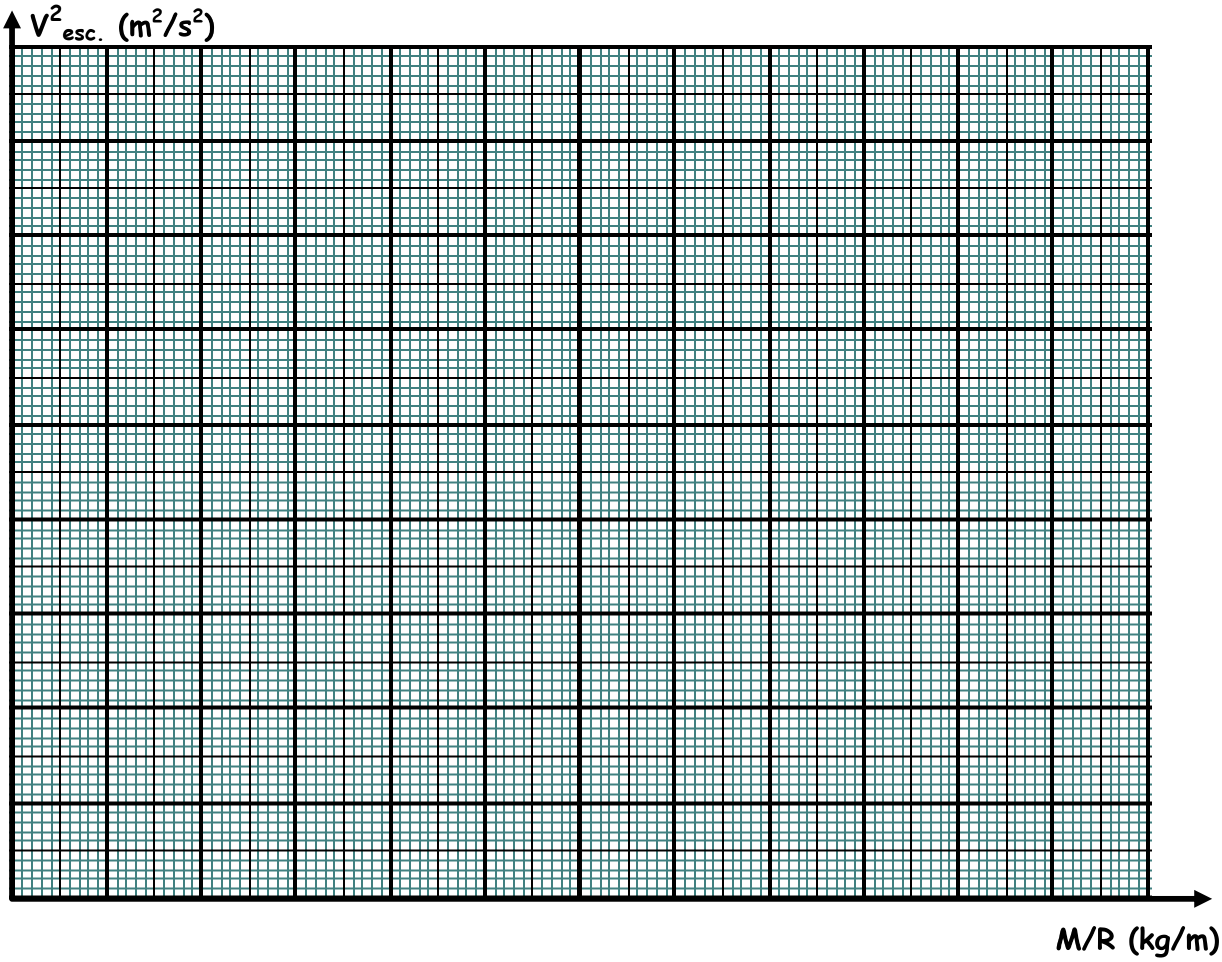
2nd activity:

Fill in the table below taking data from the application O_&_E and create the graphs $V^2_{circular}$ vs [M/R] and V^2_{escape} vs [M/R]. Consider the mass of the Earth: $M_E = 6 \cdot 10^{24}$ kg. (An easy way to take data from the application could be to launch the object from the surface of the planet and change only its radius.)

	M (10^{24} kg)	**R** (m)	**M/R** (kg/m)	$\mathbf{V_{circ.}}$ (m/s)	$\mathbf{V^2_{circ.}}$ (m^2/s^2)	$\mathbf{V_{esc.}}$ (m/s)	$\mathbf{V^2_{esc.}}$ (m^2/s^2)
1.							
2.							
3.							
4.							
5.							
6.							



After scratching the graph indicate the slope and write the value of the coefficient. Write the final equation of the circular orbit velocity as a function of the mass of the planet and the distance of the point of launch from the centre of the planet.



After scratching the graph indicate the slope and write the value of the coefficient. Write the final equation of the escape velocity as a function of the mass of the planet and the distance of the point of launch from the centre of the planet.

Evaluation

1. Write briefly what you defined and describe the method you followed.

2. In the application the object is launched horizontally. What would have happened if it was launched in another direction with the escape velocity? Explain your answer.

3. Launch an object from a height of 10^6 m over the surface of the Earth, with velocity $8 \cdot 10^3$ m/s. Define the perigee and the apogee of its orbit and find the escape velocity from these two points.

4. Somewhere in the Milky Way there is a planet with mass equal to the Earth's mass and radius $5{,}4 \cdot 10^6$ m. What is the escape velocity from a distance 1000 km from the surface of the planet? You are asked:

a) to determine it theoretically and
b) to verify with the applet.
(with this or with the opposite order)

5. Consider an object launched at a distance R from the centre of a planet. If you give several values to the mass of the planet end record the respective velocities of circular orbit and escape, you can draw graphs like the ones you created, while working on the 2nd activity.
What would the slope of the graph V^2_{escape} vs GM/R be?
What would the slope of the graph $V^2_{circular}$ vs GM/R be?

6. The radius of a planet, which has no atmosphere is 4000 km and the velocity of circular orbit as close as possible to its surface is 3 km/s. Determine the time of the motion of an object, which is let free to fall from a distance 18 m over the ground.