**Orbit and escape**
**An introductory approach to the space travels.**

1. Introduction:
It has been more than 100 years since Konstantin Tsiolkovsky proved theoretically that interplanetary travels could become doable and this kind of travels still attract the interest and imagination of every one, because of their mystery and the huge amount of know how required. With regard to a travel like this, students will be challenged to solve two theoretical problems of strategic nature: The problem of the circular orbit around a stellar body and the problem of escaping from the gravitation field of such a body. The solution comes using the laws and principles of Newton's Mechanics, but here we will use a java application, which plays the role of a planetary laboratory.

2. Age of students:
15-17 years old.

3. Aims of the unit:
It is expected that students will determine the formulas that give the circular velocity of a satellite orbit around a celestial body and the escape velocity of this body, using the proposed applet and following an essentially experimental method. At first level students will define the formulas in the type of proportionality. Furthermore, approaching in a higher level, they will define the coefficient of this proportionality changing it to equality. Through this process they will become familiar with the specific aspects of the Newtonian universal gravitation theory.

4. Presupposed knowledge:
- Laws of rectilinear and circular motion and the respective physical magnitudes.
- Newton's $2^{nd}$ law of motion.
- The law of universal gravitation.
- The law of the mechanical energy conservation in the gravitational field of a stellar body.
- Mathematical level appropriate to draw a graph and extract conclusions from a graph.

5. Description of the applet:
The applet is compatible with chrome, internet explorer or mozilla as long as java 1,5 or newer has been installed. On the computer monitor, our planet and a small body (apple), in the foreground of the vast Universe can be seen.
A) Input: Radius of a planet, distance from its surface, initial speed, the mass $M_i$ of the planet expressed in Earth's masses ($M_{earth}$) and the time unit. If the radius is $6{,}4{\cdot}10^6$ m and $M_i/M_{earth}=1$ (the default values of the application), then our globe appears. With any other setting of the radius or $M_i/M_{earth}$ values, a green planet comes on stage. All values are controlled by sliders. By clicking on a slider, the respective quantity can be adjusted with the arrows of the keyboard. The same result has the common use (dragging) of the slider.
B) Output: Instantaneous distance from the center of the planet (R) and height from its surface, instantaneous speed (v) and acceleration (g) of the moving body and time in seconds and in hours-minutes-seconds, typed on the monitor.

C) Progress: In the folder O_&_E, there is the file "start.html", which loads the application. The application starts with the Start/Stop button and the movement of the precision slider towards its right or left end. The smaller the value of the precision, the more precise the orbit, while the application develops slower. In the background of the applet “runs” the gravitation law and the magnitudes are almost real. The time unit varies, from 100 msec to 5 sec. The application accepts adjustment changes while running and then it starts from the beginning, with the new settings. Any time the moving object goes out of monitoring, the screen zooms out, so that the whole trajectory can be watched. In the case of orbiting, the application stops when the object returns to the point of launch (or falls on the planet). If the initial speed is not enough for orbiting, the application stops when the object crashes on the planet. In both cases a message with the velocity of the circular orbit, related to the specific settings, appears. Finally, if the body reaches an area where the acceleration is less than $10^{-3}$ m/s$^2$, the message “practically out of the planet's field” appears and the application stops. Simultaneously, a second message with the escape velocity related to the values adjusted, comes on the screen.

4. Way of work:

At first level students will follow a common experimental method, collect data from the interactive application and proceed these data:

Keeping the mass of the planet unchanged, they will adjust radius or / and height and will find the speed of the circular orbit at this distance from the center of the planet. Then they fill in a table R-V. Taking into account some mathematical aspects related to proportional and reversely proportional quantities, they will define the dependence of V on R. (R=r+H)

$$V_{circular} \sim R^{-1/2}.$$

In a similar way, keeping unchanged the distance from the planet's centre, students will define the dependence of V on the mass of the planet.

$$V_{circular} \sim M^{1/2}.$$

So, finally they will come to the proportionality:

$$V_{circular} \sim [M/R]^{1/2}$$

In a second higher level of approach, they will create the graph: $V^2_{circular}$ versus M/R . It is expected that the slope of the graph will be something close to G (The universal gravitation constant: $G=6{,}67\cdot 10^{-11}$ Nm$^2$/kg$^2$). The units will be defined by the comparison of the units of $V^2$ and M/R.

Finally: $V_{circular} = \sqrt{\dfrac{GM}{r+H}}$.

Working with the same technique, students will define that $V_{escape} \sim [M/R]^{1/2}$ in the first level of approach and in the second level that the coefficient is $\sqrt{2G}$.

Finally: $V_{escape} = \sqrt{\dfrac{2GM}{r+H}}$.

The two worksheets (the second one covers the higher level approach) should be given to the students. They are based on these ideas and are completed with some interesting issues, such as the first and the second cosmic velocities and the gravitational equivalency between a homogenous spherical object and a material point of equal mass located at the centre of it, as far as the total area out of the spherical object regards. The last part of each worksheet has some indicative questions for evaluation.

5. Theoretically:

A. In the circular orbit the gravitational force on the moving object has the role of the centripetal force. So:

$G\dfrac{Mm}{(r+H)^2}=\dfrac{mV_{circular}^2}{r+H}$ and finally: $V_{circular}=\sqrt{\dfrac{GM}{r+H}}$

B. In the case of escape, we start from the principle of the mechanical energy conservation. The initial velocity of the moving body, crucially enough to reach the area out of the gravitational field of the central body, in infinite distance from it, where both the potential and the kinetic energy are zero, is the "escape velocity". So:

$U_R + K_R = U_\infty + K_\infty$, therefore: $-G\dfrac{Mm}{r+H}+\dfrac{1}{2}mV_{escape}^2=0+0$. Finally: $V_{escape}=\sqrt{\dfrac{2GM}{r+H}}$.

Note: We focus on the "experimental" method with the simulation but it is proposed that pupils should work in both ways for a better comprehension.

6. Materials:

A. The application: "O_&_E"
B. An introductory video and a quick manual for the use of the application
C. Two worksheets with proposed activities and indicative questions, for students who would like to engage in the teaching of this section.
D. An extra worksheet for younger pupils.
E. The present introduction.