LENGTH OF DAY

INTERNET FILE

In this section we explain the steps that we follow to calculate the length, and they could be used as a guide to calculate manually and also to program an application like the one provided with ICT.

1) Students introduce in a tab the date for wich they want to know the length of day. In another tab they introduce the latitude and longitude of the place for which they want to make calculations $\{\phi, \lambda\}$ (latitude in degrees, longitude in hours).
{year, month, day} in number, for instance: {2011, 11, 25}.

2) The program passes the latitude the degrees to radians.

3) The program calculates the ascension recta and declination of the Sun $\{\alpha_{\oplus}, \delta_{\oplus}\}$ for that day. How?

a) The date choosen is passed to Julian Days (a system of time measurement used by the astronomy community presenting the interval of time in days elapsed since the epoch 1900 January 0.5 , because it is noon at Greenwich on 31 December 1899). The hour is forced at 12:00 a.m. So, given a {day, month, year} to calculate the Julian days we need:

Let be: 'a = (14 – month) / 12' 'y = year + 4800 – a' 'm = month + 12·a – 3'

So, 'JD[day,month,year] = day + (153·m + 2)/5 + 365·y + y/4 – y/100 + y/400 – 32045' given the actual Julian Day.

b) x is (in DJ[x)] Julian days but we want to pass this valour to Julian centuries. Let to be T the Julian centuries by means of the formula:

$$T=\frac{x-2451545}{36525} \quad \text{and} \quad U=\frac{T}{100}$$

c) When we know U and we can determine the longitude of the Sun ($\oplus$) using the following formulas:

$$Longitud_{Sun}=4.9353929+62833.1961680*U+10^{-7}*\left(\sum\left(LSun\cdot\sin\left(AlfaSun+NuSun*T\right)\right)\right)$$

being **lSun**, **alfaSun** and **nuSun** arrays of 50 elements containing values of the Sun. This value is not corrected in terms of nutation and aberration and the final solution may differ about 30 minutes with the real length of the day.

LSun = { 403406, 195207, 119433, 112392, 3891, 2819, 1721, 0, 660, 350,334, 314, 268, 242, 234, 158, 132, 129, 114, 99, 93, 86, 78, 72, 68, 64, 46, 38, 37, 32, 29, 28, 27, 27, 25, 24, 21, 21, 20, 18, 17, 14, 13, 13, 13, 12, 10,10,10, 10 }

AlfaSun = { 4.721964, 5.937458, 1.115589, 5.781616, 5.5474, 1.5120, 4.1897, 1.163, 5.415, 4.315, 4.553, 5.198, 5.989, 2.911, 1.423, 0.061, 2.317, 3.193, 2.828, 0.52, 4.65, 4.35, 2.75, 4.50, 3.23, 1.22, 0.14, 3.44, 4.37, 1.14, 2.84, 5.96, 5.09, 1.72, 2.56, 1.92, 0.09, 5.98, 4.03, 4.27, 0.79, 4.24, 2.01, 2.65, 4.98, 0.93, 2.21, 3.59, 1.50, 2.55}

NuSun = { 1.621043, 62830.348067, 62830.821524, 62829.634302, 125660.5691, 125660.9845, 62832.4766, 0.813, 125659.310, 57533.850 ,-33.931, 777137.715, 78604.191, 5.412, 39302.098, -34.861, 115067.698, 15774.337, 5296.670, 58849.27, 5296.11, -3980.70, 52237.69, 55076.47, 261.08, 15773.85, 188491.03, -7756.55, 264.89, 117906.27, 55075.75, -7961.39, 188489.81, 2132.19, 109771.03, 54868.56, 25443.93, -55731.43, 60697.74, 2132.79, 109771.63, -7752.82, 188491.91, 207.81, 29424.63, -7.99, 46941.14, -68.29, 21463.25, 157208.40}

The output of the Sun longitude $\oplus$ is in radians.

d) We calculate the oblicuity of the ecliptic using the formula:

Oblicuity = 0.4090928 - 0.0226938 * U + 10^{-7} * (-75 * U^2 + 96926 * U^3 - 2491 * U^4 - 12104 * U^5 + (446 * *cos*(NutA1(U)) + 28 * c*os*(NutA2(U))))

being: **NutA1(x)** = 2.18 - 3375.70 * x + 0.36 * x^2 and **NutA2(x)** = 3.51 + 125666.39 * x + 0.10 * x^2

e) We determinate $\delta_\oplus$ and $\alpha_\oplus$ from the equations: $\begin{cases} \sin\delta_\oplus = \sin\varepsilon \sin\oplus \\ \tan\alpha_\oplus = \cos\varepsilon \tan\oplus \end{cases}$

Remember in the last equations that the calculation of the arc.tan for the alfaSun may give you not the good value but the value in other quadrant. If you are not sure check your tangent value calculating tang with the sin and cos and checking the quadrants.

f) Students introduce ϕ in degrees but we need the latitude in radians, so we pass ϕ of degrees to radians.

4) Then we calculate $H_0 = \arccos(-\tan\phi\tan\delta_\oplus)$ where H_0 is the hour angle at sunset and $H_0 \in$ [0, 12 hours].

5) So, the sidereal time (ST) is:

$$ST_{sunset} = \alpha_\oplus + H_0 \quad \text{and} \quad ST_{sunrise} = \alpha_\oplus - H_0$$

ST, sidereal time in in the interval [0,2π].

6) Now we have to pass the sidereal time ST, to universal time UT for sunset and sunrise, the formula is:

$$UT = 0{,}9972696\,(ST - GMST_0 - \lambda) \text{ in hours,}$$

where:
ST are the values obtained above for sunset and sunrise; $GMST_0$ is the Greenwich sidereal time at 0 hours of UT a given day and λ is the longitude of the place for what we are doing calculations.
If UT results to be negative we have to reconsider calculate it again forcing to be in the interval [0,2π] doing module 2π operation.

$GMST_0$ changes from day to day and its value is given by the expression:

$$GMST_0 = 6^h41^m50^s.54841 + 8640184^s.812866T + 0^s.093104T^2 + 0^s.0000062T^3$$

where T is, the value we have calculated in paragraph 3)b).
As we have ST and λ in hours we have to pass $GMST_0$ to hours for the UT value was in hours as we wanted.

7) We calculate UT for sunset and sunrise.

$$UT_{sunset} = 0{,}9972696\,(ST_{sunset} - GMST_0 - \lambda)$$
$$UT_{sunrise} = 0{,}9972696\,(ST_{sunrise} - GMST_0 - \lambda)$$

These values give us the time of sunrise and sunset for a day and place determinated in Universal Time (UT).

8) The length of day is: $UT_{sunset} - UT_{sunrise}$

Well as to determine the time of sunrise and sunset one day at a particular location and duration of that day, we want to make a graphical representation of the variation in the height of the Sun above the horizon on the same day. So,

9) We need a change of coordinates: the equatorial coordinate to horizontal coordinate.

$$(\alpha_\oplus, \delta_\oplus) \xrightarrow{ST} (H, \delta_\oplus) \xrightarrow{\phi} (A, h)$$

where□ ST is sidereal time, H is the hour angle, ϕ is the latitude of the place, A is the azimuth and h is the elevation, altitude or height of the celestial body (in our case the Sun) on the horizon. But $H = ST - \alpha_\oplus$.

To calculate the horizontal coordinates in terms of equatorial coordinates we have to solve the system of equations:

$$\begin{cases} \sin z \sin A = \cos \delta_\oplus \sin H \\ \sin z \cos A = \cos \delta_\oplus \cos H \sin \phi - \sin \delta_\oplus \cos \phi \\ \cos z = \cos \delta_\oplus \cos H \cos \phi + \sin \delta_\oplus \sin \phi \end{cases}$$

z is the zenith distance and z=90º-h.

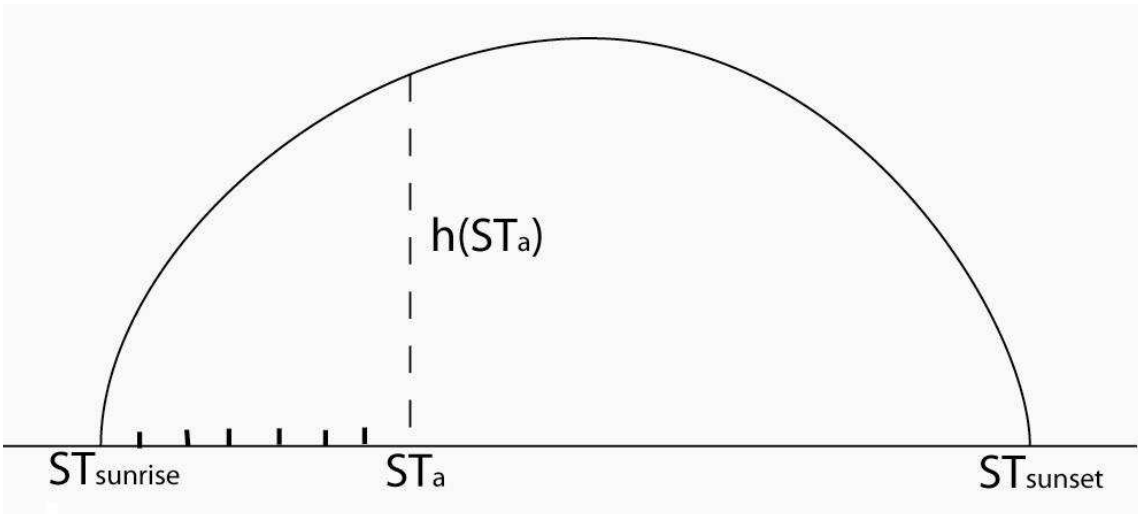
As we just want to know *h* we have to solve the only the last equation of the previous system, that is:

$$\cos(90 - h) = \cos \delta_{\text{Â}} \cos(ST - \alpha_{\text{Â}}) \cos \phi + \sin \delta_{\text{Â}} \sin \phi \, .$$

For each value of *ST* we will find a value of *h* (altitude of the Sun in this moment *ST*) (Fig.5: Graph of heights of the Sun during a day.)

If we want to draw the graph of the path of the Sun above the horizon during the day, we have to give to ST values of the interval [$ST_{sunrise}$, ST_{sunset}], that is:

$ST \in [ST_{sunrise}, ST_{sunset}]$. [Figure_Internet_1]



Graph of heights of the Sun during the day

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