

## 1| Summary

This teaching unit is about parallax measurements-how to determine distances in the universe.

Distances in space seem unreal to our students. The parallax effect is an abstract idea that is difficult to understand. Starting with outdoor hands-on activities, we measure distances using mobile devices and simulate distance measurements in the classroom, on the school campus and to the moon. In an international collaboration project we can also determine the altitude of the ISS.

- Keywords: parallax method
- Disciplines: physics, astronomy, mathematics, geometry, trigonometry
- Age level of students: $14-18$ years
- Android apps: Distance and Parallax, Stellarium, ISS

Detector, Theodolite droid, Compass, Smart Protractor, Smart Measure

- iOS apps: Stellarium, Theodolite, Angle Meter, Compass


## 2| Conceptual introduction

In most European countries astronomy is not an independent subject, but is integrated into physics. Parallax measurements and calculations are also part of mathematics. In some countries astronomy is studied in the first year of high school.

A difficult problem in teaching astronomy is how to determine the distances of planets, stars, galaxies and other objects in the sky. How can astronomers measure the distances to these objects? For objects up to about 100 light years away they use a trigonometric method to measure a visual phenomenon called the parallax effect.

Preconditions: Students must know the basic star classification system (HR diagram, colour and temperature), how stars evolve, and how astronomical distances are measured. They must be familiar with basic geometry and trigonometric functions.

## ${ }^{3}$ | What the students do

3|1 General introduction to the parallax methodmeasuring distances on the school campus


3|1|1 Experiencing the parallax effect with the thumb You can see the parallax effect in action by simply stretching out your arm in front of your face, sticking up your thumb, and selecting an object within the room. Close your left eye and position your thumb in line with the object you have selected. Now open your left eye and shut your right eye without moving your thumb, and look at your thumb again. Even though you have not moved your thumb, you will notice that it has appeared to change its position. This shift in the position of your thumb in relation to the object in the background is called the parallax effect. It happens because each of your eyes is looking at your thumb from a different position in space. This is the parallax shift. The distance between the two points is the baseline.


You can estimate the distance to an object by using the parallax shift and applying the following formula:

[^0]rill FIG.4 E: distance from the eyes, L: arm length, t: thumb position


It is important to emphasise that the distance is inversely proportional to the parallax angle. If the distance between the Earth and a star is long, the parallax angle is small; if the distance is short, the parallax angle is large.

## $3|1| 2$ Using a handmade angular measurement device

How can we easily determine the distance to close objects, e.g. in the classroom or in the school garden?

We were inspired by "PARALLAX - IT'S SIMPLE!". In this paper the authors created a cardboard device to measure the parallax effect.

As shown in fig. 5, we developed a device to measure the parallax angle. It consists of a direction-finding part (which is turnable and moveable) and a rack (baseline $=1$ metre).


EXPERIMENT 1 Learn how to measure distances in the classroom by using the parallax method

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Materials used
- Protractor (or smartphone with a compass app)
- Measuring device
D Object
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For short distances you would not need this method, but you can learn how it works and how to be accurate.

Place the object (e.g. a bottle) somewhere on a table. Put the rack a few metres away, at a right angle to the viewing direction (rack-object). Move the turnable part to the other end, focus on the object (by eye or using a laser) and measure the angle between the two lines of sight. You can calculate the distance $L$ by using the formula (FIG. 3).

## experiment 2 Use the parallax method to measure distances on the school campus

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Materials used
- Measuring device
- Protractor (or smartphone with a compass app)
- Streetlight on the school campus
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Place the measurement rack at a distance from the object. Point to the object from both ends of the rack and determine the angle between the two lines of sight. Instead of a classic protractor you can use a smartphone (with a compass app). It is rather tricky to measure the angle using a magnetic compass; make sure no iron objects are near the measurement device.

Instead of this device you can use a scale or the border of a table with a known length. The measurement will become more exact if you use a longer scale, e.g. 3 metres.
$3|1| 3$ Practice and think about the parallax method using apps
There are a lot of apps you can use to measure (short) distances, e.g. Smart Measure. They use the phone's camera. Measure some distances within the school building and compare them with the results of measuring tapes.

Using the picture, explain how the measurement works. (The camera is used to locate the object. The phone uses its $g$-sensor to determine the slope. It calculates the distance by using the known height h (input) and the angle of $90^{\circ}$ at the bottom.)

Smart Measure is an appropriate tool for measuring the height of an object and its distance from you. This telemeter measures the distance, height, width and area of an object by using trigonometry with your smartphone. It's simple to use: you only have to stand up, push the button and aim the camera at the ground, not the object, to find out the distance instantly.

The Distance and Parallax app works with our eyes and a pencil. Although it is a simple calculation tool, it convincingly shows the principle of parallax measurement.
$3 \mid 2$ Determine the distance to the moon (simulation)
The students have to simulate a parallax measurement by using data they get from apps. Principle: A virtual observation.


We need two locations on Earth on the same meridian (i.e. at the same geographical longitude), but far away - e.g. in Europe and South Africa. Then we have to measure the altitude of the moon (the angle between it and the horizon) when it passes the meridian, simultaneously from both locations.

To make the calculation easy, we choose two locations: $60^{\circ}$ North and $60^{\circ}$ South, both at the longitude of $15^{\circ}$ East. Here they are shown for the Stellarium app.

FIG.? Moon in the northern sky (upper figure) and in the southern sky (lower figure)


We have to find out the time when the moon passes the meridian. For 9 February 2014 this was 8:20 p.m. Stellarium shows the moon's altitude above the horizon. (Note that in the northern hemisphere the meridian is in the south, and in the southern hemisphere it is in the north.)


We can see that: h (north): $48^{\circ} 41^{\prime}=48.68^{\circ}$;
h (south): $9^{\circ} 48^{\prime}=9.8^{\circ}$


This gives us the following situation:

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FIG.10 E: Earth; D: distance to the moon
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The baseline $b(N-S)$ can be calculated with: $\sin 60^{\circ}=b / 2: 6,370 \mathrm{~km} \rightarrow \mathrm{~b}=11,033 \mathrm{~km}$

Adding $60^{\circ}$ to each of the altitudes, we get for the parallax angle $\pi$ : $180^{\circ}-60^{\circ}-60^{\circ}-48.68^{\circ}-9.8^{\circ}=1.52^{\circ}$.

Simplification: We calculate with a right angle $\rightarrow \tan \pi=b / D$, which gives us $D=415,786 \mathrm{~km}$. Compared with the real distance of $398,733 \mathrm{~km}$, this is $4 \%$ too far.

## 4| Cooperation option

The altitude of the ISS-an international collaboration project
The International Space Station is a very bright object that is fascinating to observe. Because it is very close to the Earth ( $300-400 \mathrm{~km}$ high), it is possible to use the parallax method to calculate the altitude of a real astronomical object.

We can estimate the altitude of the International Space Station by using angle measurements taken from two locations at the same time.

- 1. Using the app ISS Detector: Find a possible observation point at each location. Agree on the measurement time. This moment should be as exact as possible (within seconds), because the ISS is moving very fast.
- 2. Determine the air-line distance and the aberration of this line in a northerly direction.
- 3. You have to measure two angles at the same time: the altitude and the compass point (angle to the north). You can use two apps for this purpose: Smart Protractor (altitude) and the compass. Alternatively, try the Theodolite app, which gives both angles at the same time.
- 4. Share the data.
- 5. Calculate the altitude, then analyse and discuss the results. How can you calculate the altitude?


In this figure the two locations are A and B. c represents the air-line distance, $\alpha$ the aberration.
$\beta$ and $\gamma$ are the directions to the ISS (here at G), pointing to the virtual base point C. So the red triangle is located on the Earth, connecting the observation points with the base point of ISS.

Using this triangle, we can calculate the ground distances a and $b$ by using the law of sines:
$\pi=180^{\circ}-\alpha-\beta$
$c / \sin \pi=a / \sin (\beta+\alpha)=b / \sin (\gamma-\alpha)$

Then we calculate the altitude of the ISS ( f ). Here it is shown for location B. We have the ground distance a and have measured the altitude $\delta$. So we get $\mathrm{f}: \tan \delta=\mathrm{f} / \mathrm{a}$

## Example:

- Locations: Graz (A) and Naples (I). Air-line distance: 300 km , aberration to the meridian: $8^{\circ}$.
- Date: 20 February 2014. The ISS Detector shows us what we can expect.

In Graz we will see the ISS moving from west to southwest, in Naples from northwest to north.


At 7:35 p.m. we measured the angles seen in FIG. 13. The red triangle shows the ground point of the ISS at this time near Verona. Using the formula above, we determine that a is 450 km and b is 540 km .

- Altitude measurements: Graz: $56^{\circ}$, Naples: $31^{\circ}$

Using the formula above, we calculated that the altitude $f$ is about 320 km .

## 5|Conclusion

What the students are expected to gain from this experiment:

- An understanding of how to measure and calculate astronomical distances (up to a hundred light years from the Earth)
- Skills in using mobile devices to make angular measurements and calculations
- An understanding of the relationship between the variability of their measurements and the accuracy of astronomic measurements


At the end of the experiments, the students should be able to calculate the effective distance between two objects thanks to very simple instruments, such as the parallax measuring device and a smartphone app. Hands-on activities can effectively involve students in the learning process and help them gain skills in the handling of scientific tools.

Students have participated with real enthusiasm and been emotionally involved in this experimental activity. Our students also learned that in the real world trigonometry functions are used not only during maths lessons but also in GPS navigation and in surveying. Astronomers use parallax angles and other techniques for finding the distances to the stars every day. This article also suggests ways in which parallax and trigonometry functions are used in real-world applications, such as surveying, celestial navigation, and even 3D movies. It also discusses how parallax angles to stars are measured, as well as other techniques for finding the distances to the moon.

## 6|References

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- OpenStreetMap: www.openstreetmap.org


## Imprint

## TAKEN FROM

iStage 2 - Smartphones in Science Teaching available in English and German
www.science-on-stage.eu/istage2

## PUBLISHED BY

Science on Stage Deutschland e.V.
Poststraße 4/5
10178 Berlin • Germany

## REVISION AND TRANSLATION

TransForm Gesellschaft für Sprachen- und Mediendienste mbH
www.transformcologne.de

## CREDITS

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## DESIGN

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## ILLUSTRATION

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First edition published in 2014
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[^0]:    FIG.3 $L=\frac{E / 2}{\tan (\alpha / 2)}$

